ON THE INTERACTION OF CORPORATE FINANCING

AND TAX

AND THE WEIGHTED AVERAGE COST OF CAPITAL

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S. C. Myers

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ON THE INTERACTIONS OF CORPORATE FINANCING

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Stewart C. Myers[**]

I. INTRODUCTION

Everyone seems to agree that there are significant interactions between corporate financing and investment decisions. The most important argument to the contrary -- embodied in Modigliani and Miller's (MM's) famous Proposition I -- specifically assumes the absence of corporate income taxes; but their argument implies an interaction when such taxes are recognized. Interactions may also stem from transaction costs or other market imperfections.

The purpose of this paper is to present a general approach for analysis of the interactions of corporate financing and investment decisions, and to derive some of the approach's implications. Perhaps the most interesting implication is that the weighted average cost of capital formulas proposed by MM and other authors are not always correct. Except in certain special cases, a more general "Adjusted Present Value" rule should, in principle, be used to evaluate investment opportunities.

The paper is organized as follows. Section II presents the framework for my analysis, which is a mathematical programming formulation of the problem of financial management. The conditions for the optimum and the implications for corporate investment decisions are derived. In Section III, the usual weighted average cost of capital rules are

derived as special cases of the more general analysis. Section IV describes the errors that can occur if weighted average cost of capital rules are used in practice. Also, I discuss the Adjusted Present Value rule as an alternative decisionmaking tool. The last section briefly describes some topics for further work, notably in the development of programming models for overall financial planning.

It must be emphasized that this paper is not intended to catalogue or deal with all possible interactions of financing and investment decisions; in other words, there is no attempt to specify the problem of financial management in full generality. Instead, I present an approach to analyzing interactions and a specific analyses of the most important ones.

Although the paper is an exploratory step towards a general model of financial management, the analysis is nevertheless of immediate interest. As far as I know this is the first full statement of the assumptions underlying **cost of capital** concepts, and the first explicit calculation of the errors that can result if the assumptions are false.

II. BASIC FRAMEWORK

Specification of the Problem

We will consider the firm's problem in the following terms. It begins with a certain initial package of assets and liabilities. For a brand-new firm, this may be simply money in the bank and stock outstanding. For a going concern, the package will be much more complicated. Any firm, however, has the opportunity to change the characteristics of its initial

package by transactions in real or financial assets -- i.e., by investment or financing decisions.

The problem is to determine which set of current and planned future transactions will maximize the current market value of the firm. Market value is taken to be an adequate proxy for the firm's more basic objective, maximization of current shareholders' wealth.[2]

This type of problem can be approached by (1) specifying the firm's objective as a function of investment and financing decisions and (2) capturing interactions of the financing and investment opportunities by a series of constraints.

Example of the Approach

Before moving on to more general formulations, I will present a simple numerical example.

Consider a firm which has to decide how much to invest and/or borrow in the coming year. Let:

> x = New investment, in millions of dollars.
>
> y = New borrowing, in millions of dollars.

Also, assume that:

1. Available investment opportunities can absorb $1 million at most. The investment generates a perpetual stream of after-tax cash flows. Let the expected average value of these flows be C. In this case $C = .09y$.

2. Assume the market will capitalize the returns at a rate $\rho_0 = .10$. Thus, if all-equity financing is used, these assets generate a net present value of - $.10 per dollar invested.

3. New debt is limited to 40 percent of new investment.

4. The firm has $800,000 in cash available.

5. Any excess cash is paid out in dividends.

6. The additions to debt and equity are expected to be permanent.

In order to specify the objective function in the simplest possibly way, I will assume that MM's view of the world is correct. If so, it is sufficient to maximize the overall market value of the firm. V is given by[3]

$$V = V_0 + PVTS \qquad\qquad (1)$$

where V_0 = the market value of the firm given all-equity financing, and

PVTS = the present value of tax savings due to corporate debt. Dividends paid are not explicitly included in Eq. (1). Under MM's assumptions, dividend policy is irrelevant, given the firm's investment and borrowing decisions.[4]

Therefore, ψ, the increase in the market value of the firm, is -.1x + .5y. This is to be maximized subject to constraints on the amount invested ($x \leq 1$), the amount of debt issued ($y \leq .4x$) and the balance of sources and uses of funds ($x \leq y + .8$).

The solution. -- This is the linear programming problem depicted in Figure 1. It is evident from the figure that the solution is at x = 1, y = .4. The constraints on the amounts invested and borrowed are each binding at the optimum. The sources/uses constraint is not binding, however: the firm has $200,000 available for dividends.

Why does the optimal solution call for investing in a project with a negative net present value? The reason is that the project allows the firm to issue more debt, and the value of tax savings generated by

New
Assets
(millions)

Optimal
Solution

y = .4x

x = .8 + y

x = 1

Feasible
Solutions

-.1x + .5y = constant

.2

.4

.6

.8

1.0

1.2

.1   .2   .3   .4

y

New Debt
(millions)

The Problem: Max: -.1x + .5y

Subject to: $x \leqslant 1$
              $y \leqslant .4x$
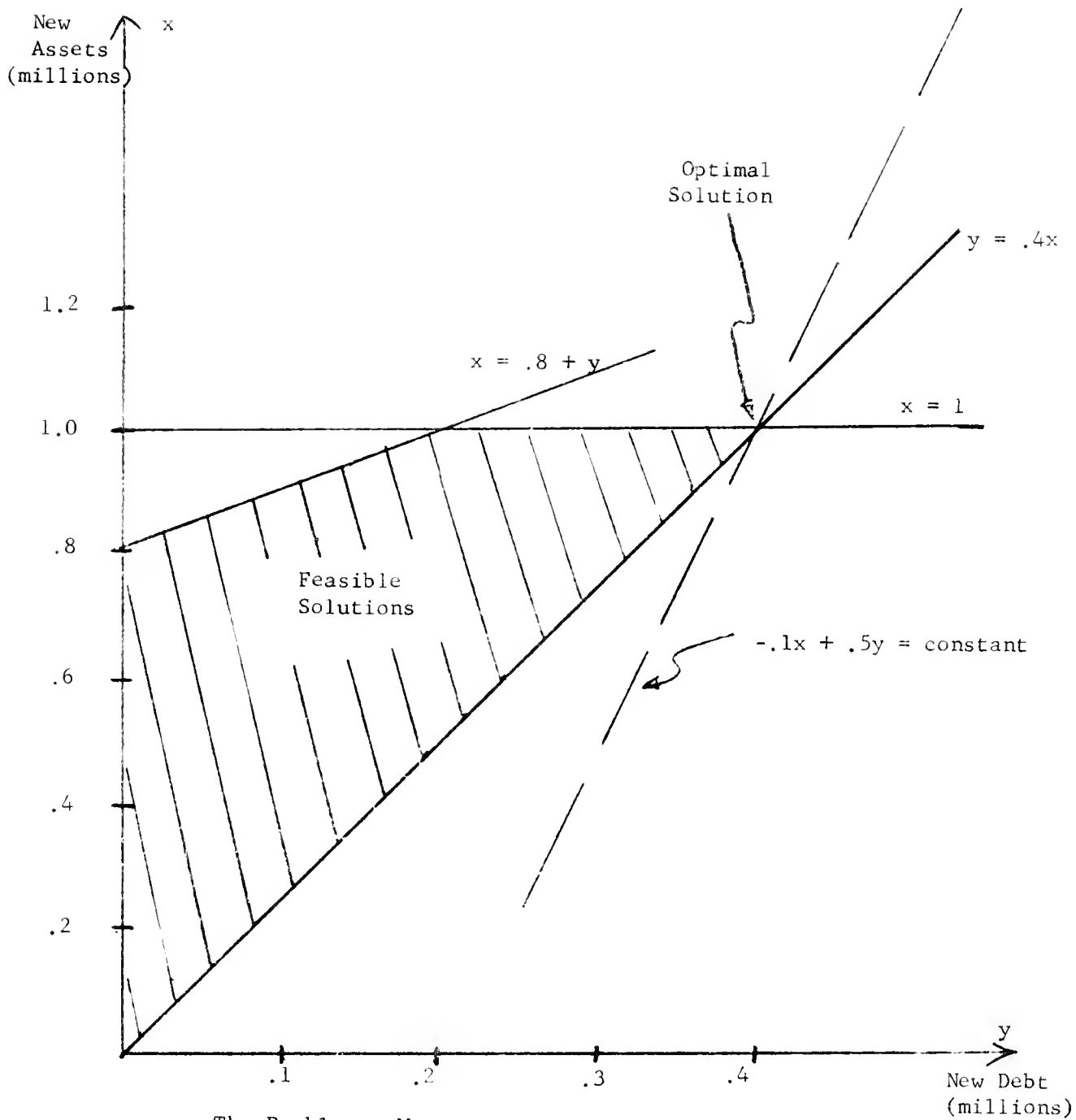 $x \leqslant y + .8$

FIGURE 1:   GRAPHICAL SOLUTION OF THE PROGRAMMING PROBLEM

the debt more than offsets the investment's inadequate return. (In fact, the optimal solution remains x = 1.0, y = .8 so long as the investment generates more than -$.20 per dollar invested. If it generates less than that, the solution becomes x = y = 0.) The debt capacity constraint thus reflects a significant interdependency between financing and investment decisions.

The solution is consistent with the normal idea that less profitable projects can be undertaken where investments are partly financed by debt -- i.e., that "the cost of capital declines as a function of financial leverage," at least for "moderate" debt levels. However, note that the investment's net present value of -$.10 per is not calculated using a weighted average cost of capital. This would be inconsistent. It would presume something about the proportion of debt financing to be used, which is not known until the problem is solved. The programming formulation is intended to reach optimal investment and borrowing decisions simultaneously. This is why the investment opportunity in the example is evaluated assuming the "base case" of all-equity financing, and the value per dollar of debt issued is then evaluated by computing the change in the objective function relative to the base case.

Effects of dividend policy. -- The sources/uses constraint is not binding in the example, and therefore does not create an interaction of financing and investment decisions. However, what if it is binding? What if the firm has, say, only $500,000 cash on hand?

At first glance the effect is to change the sources/uses constraint to x ≤ y + .5, which changes the optimal solution to x = 5/6,

y = 1/3. However, if we assume that dividend policy is irrelevant, then consistency requires that there be no restraints on new issues of equity. The constraint should really be

$$x + D \leq y + .5 + E$$

where D = dividends paid, in millions, and if dividend policy is irrelevant, then D and E have no effect in the objective function, and the constraint itself is irrelevant. Thus the optimal solution remains at x = 1, y = .4. The firm must make a $100,000 stock issue, but this is a mere detail once the investment and borrowing decisions are made.

However, in practice there would be transaction costs associated with the stock issue. These costs would have to be subtracted from the objective function,[5] and the sources/uses constraint would become relevant and binding. The solution values for x and y will clearly be affected, if the transaction cost is large enough.

Summary. - The example shows how a firm's optimal investment and financing decisions can be reached simultaneously in a mathematical programming format. It also shows how constraints on debt capacity and sources and uses of funds can embody significant interactions between financing and investment decisions.

A More General Formulation

Consider a firm which has identified a series of investment op-portunities. It must decide which of these "projects" to undertake.[6] At the same time it wishes to arrive at a financing plan for the period t = 1, 2, . . ., T. The financing plan is to specify, for each period the planned

stock of debt outstanding, cash dividends paid, and the net proceeds

from issue of new shares.

Let: x_j = proportion of project j accepted.

y_t = stock of debt outstanding in t.

D_t = total cash dividends paid in t.

E_t = net proceeds from equity issued in t.

C_{jt} = expected net after-tax cash inflow of project j in t, with net outflow (i.e. investment) represented by $C_{jt} < 0$.

Z_t = debt capacity in t, defined as the limit on y_t. Z_t depends on firm's investment decision.[7]

Also, let ψ equal the change in the current market value of the firm,

evaluated cum dividend at the start of period t = 0. In general, ψ is a

function of the x's, y's, D's and E's.

The problem is to maximize ψ, subject to:

$$\emptyset_j = x_j - 1 \leq 0, \qquad j = 1, 2, \ldots, J. \qquad (2a)$$

$$\emptyset_t^F = y_t - Z_t \leq 0, \qquad t = 0, 1, \ldots, T. \qquad (2b)$$

$$\emptyset_t^C = -x_j C_{jt} + y_t - y_{t-1}[1 + (1-\tau)r] + D_t - E_t = 0$$

$$t = 0, 1, \ldots, T. \qquad (2c)$$

The borrowing rate, r, is assumed constant for simplicity, as is the cor-

porate tax rate τ. In general, r will be a function of the other variables.

Eqs. (2) define the nature of the interactions between the firm's

financing and investment decisions. The effects of these interactions can

be better understood by examining the conditions for the optimal solution.

Conditions for the Optimum. -- In order to simplify notation

define $A_j \equiv \delta\psi/\delta x_j$, $F_t \equiv \delta\psi/\delta y_t$ and $Z_{jt} = -\delta\emptyset_t^F/\delta x_j$. Also, note that each

of the following equals 1: $\delta\psi\emptyset_j/\delta x_j$, $\delta\emptyset_t^F\delta y_t$ and $\delta\emptyset_t^C/\delta y_t$. Finally, note that

$\delta\emptyset_t^C/\delta x_j = -C_{jt}$. The shadow prices are λ_j for $\emptyset_j$, λ_t^F for $\emptyset_t^F$ and λ_t^C for

$\emptyset_t^C$.

With these simplifications, the conditions for the optimum can

be written as follows.

$$A_j + \sum_{t=0}^{T} \left[\lambda_t^F Z_{jt} + \lambda_t^C C_{jt} \right] - \lambda_j \leq 0. \tag{3b}$$

For debt in each period, $t = 0, 1, \ldots, t,$

$$F_t - \lambda_t^F + \lambda_t^C - [1 + (1-\tau)r] \lambda_{t+1}^C \leq 0.$$

For dividends in each period,

$$\delta\psi/\delta D_t - \lambda_t^C \leq 0. \tag{3c}$$

For equity issued in each period,

$$\delta\psi/\delta E_t + \lambda_t^C \leq 0. \tag{3d}$$

In each of these equations a strict equality holds if the corresponding

decision variable is positive in the optimal solution.

Eq. (3a) is particularly interesting because it states the conditions for accepting or rejecting an investment project at the margin. The condition is that project j's "Adjusted Present Value" (APV_j) be positive, where

$$APV_j = A_j + \sum_{t=0}^{T} [\lambda_t^F Z_{jt} + \lambda_t^C C_{jt}]. \qquad (4)$$

If the project is accepted, then $APV_j = \lambda_j$. If it is rejected then APV_j is negative and $\lambda_j = 0$.

The term adjusted present value is used because in the optimal solution A_j, the project's direct contribution to the objective, is "adjusted for" the project's side effects on other investment and financing options. The side effects occur because of the project's effects on the debt capacity and sources/uses constraints.

The practical implications of the adjusted present value concept are discussed later in the paper. At this point the conditions for the optimum should be examined more closely.

Effects of financial leverage when dividend policy is irrelevant. -- Suppose that dividend policy is irrelevant, in the sense that $\delta\psi/\delta E_t = \delta\psi/\delta D_t = 0$ for all T. Then $\lambda_t^c = 0$, from Eqs. (2c) and (2d).

Also, assume that $\delta\psi/\delta y_t$ is positive -- which is realistic, given the tax deductability of debt, regardless of whether one agrees with MM. Then the constraints $\emptyset^F$ will always be binding, Eqs. (3b) will be strict equalities, and $\lambda_t^F = F_t$ for all t.

Substituting in Eq. (4),

$$APV_j = A_j + \sum_{t=0}^{T} Z_{jt}F_t \qquad (5)$$

Eq. (3) implies that APV_j, the contribution of a marginal unit of project j to the firm's value, is measured by A_j, the "intrinsic" value of the project at the margin plus the present value of the additional debt the project supports.

Effects of dividend policy. -- In practice, however, dividend policy will not be completely irrelevant. At very least, $\delta\psi/\delta E_t$ will be negative because of transaction costs associated with stock issues. It is not clear whether $\delta\psi/\delta D_t$ is positive, negative or zero in real life.[8]

Suppose that the optimal solution calls for an equity issue in a period t. Then $\lambda_t^c = -\delta\psi/\delta E_t$ and $\lambda_t^c > 0$. Examination of Eq. (3a) shows that this is reflected in the optimal solution in two ways. First, project

j is penalized if $C_{jt} < 0$. On the other hand, the project is relatively more attractive if $C_{jt} > 0$: in this case the project generates funds and this reduces the need for a stock issue. Second, if the project contributes to debt capacity in t, this in turn reduces the need for the stock issue. This is evident in Eq. (2b), which shows that λ_t^F, the marginal value of debt capacity in t, depends on λ_t^c as well as on $\delta\psi/\delta y_t$.

The same type of interactions exist if dividends are paid in t and $\delta\psi/\delta D_t \neq 0$. However, the direction of the effect on any specific project is less clear. Eq. (3c) implies that λ_t^c may be positive or negative, depending on the sign of $\delta\psi/\delta D_t$.

Summary. -- In general, Eqs. (2a) through (2b) show that financing and investment decisions are related in a relatively complex way if debt and/or dividend policies are not irrelevant. The interrelationships exist for two reasons. First, corporate debt is limited, and the limit depends on the firm's assets. Second, sources and uses of funds have to balance. The addition of other types of constraints would create still more complicated interdependencies.

III. A REEXAMINATION OF THE WEIGHTED AVERAGE COST OF CAPITAL CONCEPT

Introduction and Definitions

It is generally accepted, at least in theoretical circles, that investment projects ought to be evaluated on a "DCF," or discounted cash flow, basis. This is done by one of two rules. The first is to compute project j's internal rate of return, R_j, from the formula

$$\sum_{t=0}^{t} \frac{C_{tj}}{(1+R_j)^t} = 0 \tag{6}$$

and to accept the project if R_j exceeds ρ_j^*, the "cost of capital" for j. The second rule is to compute the net present value of j's cash flows, discounted at ρ_j^*, and accept j if this figure is positive. Thus, j is accepted if

$$NPV_j = \sum_{t=0}^{T} \frac{C_{jt}}{(1+\rho_j^*)^t} > 0. \tag{7}$$

In either rule, ρ_j^* is the "hurdle rate" or minimum acceptable expected rate of return.

Comparing Eq. (4) to (6) and (7), it is evident that NPV_j and APV_j are intended to measure the same thing: the net contribution of j to the market value, taking account of the interactions of j with other investment and financing opportunities. There is always some value of ρ_j^* which will insure that $NPV_j = APV_j$, or that

$$\sum_{t=0}^{T} \frac{C_{jt}}{(1+\rho_j^*)^t} = A_j + \sum_{t=0}^{T} [\lambda_t^F Z_{jt} + \lambda_t^c C_{jt}] . \tag{8}$$

The problem is, how should ρ_j^* be computed, if not directly from Eq. (8)?

Of the many procedures for calculating ρ_j^*, two are of particular interest. The first is MM's. They propose[)]

$$\rho_j^* = \rho_{oj}(1 - \tau L), \tag{9}$$

where: ρ_{oj} = The appropriate discount rate assuming all-equity financing;

$\quad\quad\tau$ = The corporate tax rate, and

$\quad\quad$L = The firm's "long-run" or "target" debt ratio.

MM interpret ρ_{oj} as the rate at which investors would capitalize the firm's expected average after-tax income from currently-held assets, if the firm were all-equity financed.[10] This would restrict application of the formula to projects whose acceptance will not change the firm's risk characteristics. (However, we will see that this is an unnecessarily narrow interpretation of the MM formula.)

The second proposed formula is:

$$\rho_j^* = (1 - \tau)\, r\frac{B}{V} + k\frac{S}{V} \tag{10}$$

where: r = the firm's current borrowing rate;

$\quad\quad$k = "the cost of equity capital" -- that is, the expected rate of return required by investors who purchase the firm's stock;

$\quad\quad$B = market value of currently outstanding debt;

$\quad\quad$S = market value of currently outstanding stock, and

$\quad\quad$V = B + S, the total current market value of the firm.

I will refer to Eq. (10) as the "textbook formula," for lack of a better name. (The formula, or some variation on the same theme, appears in nearly all finance texts.)[11] It is not necessarily inconsistent with the MM formula, but it is recommended by many who explicitly disagree with MM's view of the world.

The task now is to determine what assumptions are necessary to derive Eqs. (9) and/or (10) from Eq. (4), the general condition for the optimal investment decision. I will present a set of sufficient condi-

tions, and then argue that, in most cases, the conditions are necessary as well.

Derivation of the MM Cost of Capital Rule

If MM's view of the world is correct, then the value of the firm will be V_0, the value of the firm assuming all-equity financing, plus the present value of tax savings due to debt financing actually employed. Dividend policy is irrelevant. Assuming this view is correct, the object-ive function in the mathematical programming formulation is:

$$\psi = \Delta V_0 + \sum_{t=0}^{T} y_t F_t,$$

where

$$F_t = \frac{r\tau}{(1 + r)^{t+1}} . \tag{11a}$$

That is, F_t is $r\tau$, the tax saving per dollar of debt outstanding in t, discounted to the present. (It is assumed that the interest is paid at $t=1$.) Eq. (11a) follows from Eq. (1).

Second, assume that

$$A_j = C_j/\rho_{oj} - I_j \tag{11b}$$

There are two ways of interpreting Eq. (11b). One is to say that project j is expected to generate a constant, perpetual stream of cash returns.

If $C_{jt} = C_j$, a constant for $t - 1, 2, \ldots, \infty$, then Eq. (11b) simply states the project's net present value when discounted at ρ_{oj}, the "appropriate rate" for j given all-equity financing.

However, MM interpret C_j as the expectation of the <u>mean</u> of the series $\hat{C}_{j1}, \hat{C}_{j2}, \ldots, \hat{C}_j\infty$. This does not require that C_{jt} is constant. On the other hand, one must impose conditions to insure that this mean is finite.

The reader may choose the interpretation he likes best. The form of the argument to follow is not affected.

The third assumption is that undertaking project j does not change the firm's risk characteristics of the firm's assets. That is,

$$\rho_{oj} = \rho_o, \tag{11c}$$

where ρ_0 is the firm's cost of capital given all-equity financing.

Fourth, assume that project j is expected to make a permanent and constant contribution to the firm's debt capacity:

$$Z_{jt} = Z_j, \qquad t = 1, 2, \ldots, \infty. \tag{11d}$$

Finally assume

$$Z_j = LI_j, \tag{11e}$$

where L is the long-run "target" debt ratio which applies to the firm overall. Eq. (11e) implies that adoption of project j will not change this target.

Rewriting Eq. (4) using Eqs. (11a) through (11e), we have:

$$APV_j = \frac{C_j}{\rho_o} - I_j + LI_j \sum_{t=0}^{\infty} F_t$$

$$= \frac{C_j}{\rho_o} - I_j + LI_j \tau \qquad (12)$$

The cost of capital is the project's internal rate of return (C_j/I_j) when $APV_j = 0$. Eq. (12) implies that this is

$$\rho_j^* = C_j/I_j = \rho_o(1 - \tau L),$$

which is MM's formula.

Extension of MM's Result to Projects of Varying Risk

Let us make one further assumption, that V_o is a linear function of the present values of accepted projects:

$$V_o = \sum_{j=1}^{J} x_j A_j, \qquad (11f)$$

$$A_j = \sum_{t=0}^{T} \frac{C_{jt}}{(1 + \rho_{oj})^t}$$

where ρ_{oj} is the discount rate specific to the risk characteristics of j's type. A_j may be interpreted as the market value of j if the project could be divorced from the firm and financed as a separate unleveraged enterprise.

Eq. (11f) assumes that projects are risk-independent, in the sense that there are no statistical relationships among projects' returns

such that some combinations of projects affect stock price by an amount different than the sum of their present values considered separately. In particular, risk-independence implies that there is no advantage to be gained by corporate diversification.

I have shown elsewhere that risk-independence is a necessary condition for equilibrium in perfect security markets.[12]

Eq. (11f) also assumes that projects are "physically independent" in the sense that there are no causal links between adoption of project j and the cash returns to other projects -- that is, it rules out "competitive" or "complementary" projects.[13] Such interactions make it impossible to specify an unique hurdle rate for project j, since the minimum acceptable rate of return on j may depend on whether or not other projects are accepted. However, I am not concerned with this problem in this paper.

Let us adopt Eq. (11f) and drop Eqs. (11c) and (11e). We can recalculate the minimum acceptable rate of return on the project.

$$\rho_j^* = \rho_{oj} \left(1 - \tau Z_j/I_j\right) \tag{13}$$

This has the same form as Eq. (9) but it is not restricted to projects within a single risk class. In this case it is not plausible to identify Z_j/I_j, project j's <u>marginal</u> contribution to debt capacity, with L, the firm's overall target capitalization ratio. Presumably Z_j/I_j will be more or less than L, depending on the risk or on other characteristics of the project in question.

In short, MM's formula can be extended to independent projects which differ in risk and in their impact on the firm's target debt ratio.

What if Investment Projects are not Perpetuities?

So far we have established that Eqs. (11a, b, d and f) are sufficient for the generalized MM formula, Eq. (13). Eqs. (11a) and (11f) are clearly necessary as well. But what about (11b) and (11d) which require all projects to be perpetuities?

In general, they are necessary: Eq. (13) does not give the correct "hurdle rate" for projects of limited life. (The question of whether the resulting errors are serious is taken up in the next section.)

This can be shown by a simple example. Consider a point-input, point-output project requiring an investment of I_j and offering an expected cash flow of C_{j1} in $\tau = 1$, and $C_{jt} = 0$ for $t > 1$. Assume $\rho_{oj} = \rho_o$ and $Z_{j1} = LI_j$ (and, of course, $Z_{jt} = 0$ for $t > 1$). Then

$$APV_j = \frac{C_{1j}}{1 + \rho_o} - I_j + LI_j \left(\frac{r\tau}{1 + r}\right)$$

The internal rate of return on the project is given by

$$R_j = \frac{C_{\cdot j}}{I} - 1.$$

The cost of capital is given by R_j when $APV_j = 0$. Thus

$$\rho_j^* = \rho_o - Lr\tau \left[\frac{1 + \rho_o}{1 + r}\right]. \tag{14}$$

Eqs. (13) and (14) are equivalent only in the uninteresting case of $\rho_o = r$.

The Textbook Formula

Let us reconsider Eq. (10):

$$\hat{\rho}_j^* = r(1 - \tau)B/V + k(S/V) \tag{10}$$

The $\hat{\ }$ is used temporarily to indicate a proposed value for the cost of capital, the true value being given by Eq. (8).

Probably it is intuitively clear from the foregoing that $\hat{\rho}_j^* = \rho_j^*$ only under very restrictive assumptions. First, let us assume that Eqs. (11a) through (11e) hold.[14] Also, assume that

$$V_o = \frac{C}{\rho_o} . \tag{15a}$$

That is V_o, the current market value of the firm if it were all equity financed, is found by capitalizing the firm's after-tax operating income at ρ_o. C is, of course, calculated assuming all-equity financing. Also, Eq. (15a) presumes $C_t = C$, $t = 1, 2, \ldots, \infty$.[15]

Finally, assume

$$L_j = B/V \tag{15b}$$

This implies that the firm is already at its target debt ratio.

Note that Eqs. (15a) and (15b) constrain the initial characteristics of the firm's assets and financing mix, whereas the assumptions underlying MM's cost of capital formula relate only to the marginal effects of adopting the project in question.

Now the task is to show that $\hat{\rho}_j^* = \rho_j^*$ under assumptions (11a-e) and (15a-b). Note first that the sum of (1) payments to bondholders

and (2) earnings after interest and taxes is

$$rB + kS \;=\; C + \tau B,$$

so that,

$$C \;=\; r(1 - \tau)B + kS \tag{16}$$

and

$$V \;=\; \frac{C}{\hat{\rho}_j^{\,*}} \;. \tag{17}$$

Eqs. (1) and (15a) imply:

$$V \;=\; \frac{C}{\rho_o} + \tau B \;. \tag{18}$$

We combine Eqs. (17) and (18) and solve for $\hat{\rho}_j^{\,*}$:

$$\hat{\rho}_j^{\,*} \;=\; \rho_o\left(1 - \tau B\left(\frac{\hat{\rho}_j^{\,*}}{C}\right)\right)$$

But $\hat{\rho}_j^{\,*}/C = 1/V$, and $B/V = L_j$, so

$$\hat{\rho}_j^{\,*} \;=\; \rho_o(1 - \tau L_j)$$

which was previously demonstrated to be the correct value.

Thus we have shown that the textbook formula gives the correct cutoff rate for projects under a long list of assumptions, one of which is that MM are correct. However, the formula is correct even if MM are wrong, providing the other assumptions hold. If MM are wrong, then

$$V \;=\; \frac{C}{\rho_o} + B \sum_{t=0}^{\infty} F_t \tag{18a}$$

where F_t reflects not only the present value of tax savings but also the impact of any relevant market imperfections. Then it is readily shown that the true cost of capital is

$$\rho_j^* = \rho_o(1 - L_j \sum_{t=1}^{\infty} F_t) \qquad (19)$$

Proceeding is before, we observe

$$V = \frac{C}{\rho_o} + B \sum_{t=0}^{\infty} F_t = \frac{C}{\rho_j^*} \; .$$

Solving for $\hat{\rho}_j^*$ we find it to be the value given by Eq. (19).

To summarize, the textbook formula gives the correct hurdle rate if:

1. The project under consideration offers a constant, perpetual stream of cash flows, and is expected to make a permanent contribution to debt capacity.

2. The project does not change the risk characteristics of the firm's assets.

3. The firm is already at its target debt ratio, and adoption of the project will not lead the firm to change that ratio.

4. The firm's currently-held assets are expected to generate a constant after-tax cash flow C per annum. This stream is expected to continue indefinitely.

The last of these assumptions may be surprising. We know from Eq. (8) that the true cost of capital ρ_j^* does not depend on the pattern

of expected cash flows offered by the firm's existing assets. Why should the observed value $\hat{\rho}_j^*$ depend on this pattern?

It can be readily shown that the pattern does matter. Let us assume that the life of the firm's existing assets will end at the close of $t = 1$. Retain all the other assumptions for the textbook formula, and assume MM are right. We must thus replace Eq. (15a) with

$$V_o = \frac{C_1}{1 + \rho_o} \tag{20}$$

and from Eqs. (1) and (15b),

$$V = V_o + FVTS = \frac{C_1}{1 + \rho_o} + \frac{\tau r L V}{1 + r} \tag{21}$$

Observe that $rB + kS$, the total return received by stock and bondholders is equal to $rB + kS = C_1 + \tau rB - V$. This implies

$$r(1 - \tau)B + kS = C_1 - V.$$

Thus:

$$V = \frac{C_1}{1 + \rho_o} + \frac{LV\tau r}{1 + r} = \frac{C_1 - V}{\hat{\rho}_j^*}$$

Now we can solve for $\hat{\rho}_j^*$:

$$\hat{\rho}_j^* = \rho_o - L\tau r \left(\frac{1 + \rho_o}{1 + r}\right) . \tag{22}$$

We may conclude two things. First, the pattern of expected cash flows offered by the firm's existing assets does affect the observed value $\hat{\rho}_j^*$.

Second, the value for $\hat{\rho}_j^*$ in Eq. (22) is the correct value ρ_j^* for a one-period project. Compare Eq. (14). This leads to the conjecture that if the stream of expected cash flows offered by the project is proportional over time to the cash flows of the firm's existing assets, then $\rho_j^* = \hat{\rho}_j^*$. But I have not yet shown that the conjecture is true in general.

However, if $c_t = \gamma C_t$, where γ is a constant, then we hardly need worry about the cost of capital. It suffices to determine whether $I_j/V < \gamma$, where I is the initial investment required to undertake the project.

Summary

Table 1 summarizes the necessary and sufficient conditions for the derivation of MM's cost of capital formula, the generalized MM formula, and the textbook formula. I believe this is the first full statement of these conditions.

Obviously, these conditions are quite stringent, particularly in the case of the textbook formula. The next section considers whether serious errors result when the conditions do not hold.

IV. HOW ROBUST ARE THE WEIGHTED AVERAGE COST OF CAPITAL FORMULAS?

Introduction

The derivation of a cost of capital, ρ_j^*, for practical use involves two steps. The first is to measure the ρ_{oj}'s, the market opportunity costs of investing in assets of different levels of risk. The second is to adjust these opportunity costs to reflect the tax effects of debt financing, transaction costs of external financing, etc. These two steps are explicit in the MM cost of capital formulas and implicit in the textbook formula.

The difficulties in step (1) are notorious. My experience suggests that the confidence limit on empirical and/or subjective estimates of ρ_{oj} is at least a percentage point under the most favorable conditions. That is, if the best estimate is 10 percent, it is hard to reject 9.5 or 10.5 percent as an equally plausible estimate. It may not be possible to reject even 9 or 11 percent.

Table 1

NECESSARY AND SUFFICIENT CONDITIONS FOR

COST OF CAPITAL FORMULAS

| | | Formula | | |
| | | | Generalized | |
(Equation)	Condition	MM	MM	Textbook
(11a)	Leverage irrelevant except for corporate income taxes	x	x	
(11b)	Investment Projects are perpetuities	x	x	x
(11c)	Project does not change firm's risk characteristics	x		x
(11d)	Project makes a permanent contribution to debt capacity	x	x	x
(11e)	Acceptance of project does not lead to shift of target debt ratio	x		
(11f)	Risk-independence	n.a.*	x	n.a.*
(15a)	Firm's assets expected to generate a constant and perpetual earnings stream			x
(15b)	Firm is already at target debt ratio			x

*n.a. = not applicable.

The uncertainty about the accuracy of step (1) estimates implies a certain tolerance for minor errors in step (2). How serious can these errors be, considered relative to the possible errors in step (1)? The purpose of this section is to begin exploring this question.

There are eight distinct assumptions listed in Table 1. Any one or any combination of them could be violated in practice. It is not feasible at this time to compute the error for all possible cases. Instead, I will focus on assumptions (11b) and (11d), which require that the project being considered is expected to make a permanent contribution to the firm's earnings and debt capacity. These are the only assumptions necessary for all three cost of capital rules.

The decision to concentrate on (11b) and (11d) was based on several points concerning the other assumptions.

1. Assumptions (11a) and (11f) were not considered because they may well hold in fact. The empirical evidence to date does not lead to rejection of the MM and risk-independence hypotheses, and a strong theoretical case can be made for them.[16]

2. Assumptions (11c) and (11e) are not necessary for the generalized MM formula. It is clear, of course, that substantial errors can result if either assumption is violated and either the original MM or textbook formula is used. But the extent of the error can be readily estimated by comparing the rate obtained from the original MM formula or textbook formula with the generalized MM formula.

Note that if (11c) does not hold, (11e) is not likely to hold either. A low-risk project will probably also make a large

contribution to the firm's debt capacity.

3. Assumptions (15a) and (15b) were not analyzed explicitly because
    the results of violating them will be similar in magnitude to
    the results of violating (11b) and (11d) respectively.

Effects of Expected Project Life

How great is the error when the MM or weighted average cost
of capital rules are used to evaluate projects of limited life?

I will start with an extreme case, by comparing the cost of
capital obtained via the MM rule with the true cost of capital for
a one-period project.

Remember that the MM formula generates a proposed value $\hat{\rho}^*_j$,
given by

$$\hat{\rho}^*_j = \rho_{oj}(1 - \tau L). \tag{9}$$

The correct value is

$$\rho^*_j = \rho_{oj} - L r \tau \left(\frac{1 + \rho_{oj}}{1 + r} \right). \tag{14}$$

For simplicity, we will omit the j's henceforth.

Comparing Eqs. (9) and (14), it is clear that $\rho^* > \hat{\rho}^*$ for
reasonable values of L and ρ_o. The error, E, is

$$E = \rho^* - \hat{\rho}^* = L\tau \left[\rho_o - r \left(\frac{1 + \rho_o}{1 + r} \right) \right] \tag{23}$$

From this we see that $\delta E/\delta L > 0$ and that

$$\frac{\delta E}{\delta \rho_o} = L\tau \left(1 - \frac{r}{1+r}\right) > 0.$$

The error is highest for high-risk projects that can be heavily debt financed.

Table 2 consists of values of E computed for values of ρ_o from 8 to 25 percent and for debt ratios of 10 to 60 percent. The errors range from .1 percent to more than 5 percent.

The errors shown in the bottom right of the table are serious, even granting the fuzziness of the ρ_o estimates. However, they probably would occur only rarely. If the project in question is really risky enough to call for ρ_o = .25, then it is doubtful that it could make a 60 percent contribution to the firm's debt capacity.

It is more realistic to look at the figures in the center, top right and bottom left of the table as indications of the typical error for short-run projects. These errors are on the order of one percentage point. The reader can judge for himself whether this is serious. Serious or not, it is clear that use of the usual weighted average rules result in an artificial preference for short-lived projects.

Evidently the error will be smaller, the longer the life of the project under consideration. Take a ten-year opportunity requiring investment I at t = 0 and offering a constant expected cash return for t = 1, 2, . . ., 10. Table III shows the error, E, of the MM or text-book formulas if applied to such a project. The errors are smaller but follow the same pattern shown in Table 2.

The figures in Table 3 were obtained by first calculating the project's APV by the procedure described in the Appendix, and then obtaining ρ^* from Eq. (8). The error is the difference between this ρ^* and MM's proposed value (Eq. (9)). The specific project investi-

Table 2

ERROR IN MM COST OF CAPITAL FORMULA

FOR ONE-PERIOD PROJECT

ρ_0, Cost of Capital for all-equity financing	Target Debt Ratio					
	.1	.2	.3	.4	.5	.6
.08	.001	.002	.003	.004	.005	.006
.10	.002	.004	.006	.008	.009	.011
.12	.003	.006	.008	.011	.014	.017
.16	.005	.009	.014	.018	.024	.028
.20	.007	.013	.020	.026	.033	.040
.24	.008	.017	.025	.034	.043	.051

Note: The risk-free rate is assumed to be $r = .06$, and the tax rate is assumed to be $\tau = .5$.

gated was $C_o = -\$1000$ and $C_t = 250$, $t = 1, 2, \ldots, 10$. However, the specific numbers used are not important, as it can be shown that the true cost of capital ρ^* is independent of C_o and the C_t's, providing that the <u>pattern</u> over time of cash flows $C_t (t \geq 1)$ is held constant. The proof is given in the Appendix.

This is an interesting and important result. Usually the cost of capital is calculated as a <u>minimum</u> acceptable rate of return -- i.e., as the project rate of return when $APV_o = 0$. It is not intuitively clear that the minimum rate can also be used to calculate the value of projects of more or less than minimal profitability. But it can be so used.

Table 3

ERROR IN MM COST OF CAPITAL FORMULA

FOR TEN-PERIOD PROJECT

ρ_o, Cost of Capital for all-Equity Financing	Target Debt Ratio					
	.1	.2	.3	.4	.5	.6
.08	.000	.001	.001	.001	.002	.002
.10	.001	.002	.003	.004	.006	.007
.12	.002	.004	.006	.007	.009	.011
.16	.003	.006	.010			
.20	.004	.009				
.24	.006					

Note: The risk-free rate is assumed to be $r = .06$, and the tax rate is assumed to be $\tau = .05$.

The Weighted Average Cost of Capital vs. the APV Rule

The first and clearest implication of this paper is that the textbook and MM cost of capital rules have to be used with extreme care. Of course I am not the first to recognize this;[17] but the point is worth reiterating because the assumptions underlying these rules are sometimes not emphasized.

The second and more original implication is that even the generalized MM rule leads to a biased evaluation of all but long-lived projects, and therefore that care is called for in its use also.

This is not to say that the weighted average cost of capital formulas should be discarded. They are useful rules of thumb -- in fact they should be still more useful in the future now that their assumptions are more clearly delineated.

Nevertheless an alternative is needed for cases in which one or more of the assumptions underlying the generalized MM formula are seriously violated and for cases in which additional complicating elements -- e.g., transaction costs of financing -- are introduced.

The natural choice for an alternative rule is to accept project j if its adjusted present value is positive, i.e. if:

$$APV_j = A_j + \sum_{t=0}^{T} Z_{jt} F_t > 0 \qquad (5)$$

In the event that dividend policy is relevant and/or there are significant transaction costs in new external financing, the criterion should be expanded to:

$$APV_j = A_j + \sum_{t=0}^{T} [Z_{jt}\lambda_t^F + C_{jt}\lambda_t^C] > 0 \tag{4}$$

For example, suppose that the firm plans to issue stock in t=1 regardless of whether project j is adopted. The transaction cost is $.05 per additional dollar issued.

Suppose the project requires a $1,000,000 investment but generates immediate debt capacity of $200,000. What is the effect on APV_j?

In terms of Eq. (4), we have $Z_{jo} = 200,000$ and $E_{jo} = -1,000,000$. The transaction cost implies $\lambda_t^C = .05$ and that λ_F^t decreases by .05. Thus the transaction cost shifts APV_j by $.05(200,000 - 1,000,000) = -\$40,000$.

This will also increase ρ_j^*. The extent of increase will depend on several things, including the duration of the project. A short-lived project will have to earn a very high rate of return to "pay off" the transaction cost before it expires.

Summary. -- The adjusted present value rule has both an advantage and a disadvantage for practical purposes. The advantage stems from the rule's generality: it is not bound to particular assumptions about project duration, transaction costs, target debt ratios, the effects of dividend policy, etc. The disadvantage is complexity.

The procedure for actually calculating APV should be clear from its general definition, Eq. (4), and from the Appendix.

V. CONCLUDING COMMENTS

In principle corporate investment and financing decisions should be made simultaneously, since the decisions interact in important ways. This paper presents a framework in which the interactions can be analyzed. Further, the framework has been used to evaluate the most widely accepted weighted average cost of capital formulas, and to derive a more general and flexible capital budgeting rule.

There are other uses for the framework. Specifically, it is possible -- given some additional assumptions -- to develop a linear programming model that can be of direct assistance to management responsible for overall financial planning. This model will be described in a paper written jointly with Professor G. A. Pogue.

This is not to say that the framework covers all important aspects of corporate financial management. The dynamic or sequential aspects of investment and financing decisions are not treated, for example.

Calculation of Adjusted Present Value

Calculating a project's adjusted present value turns out to be a moderately complex task. The problem is that APV_{jo}, adjusted present value of project j as of $t = 0$, depends on estimated values of APV_{jt} for later periods. If the horizon is $t = T$, we have to calculate $APV_{j,T-1}$, $APV_{j,T-2}$, etc., and then finally APV_{jo}.

We begin with Eq. (5):

$$APV_{jo} = A_{jo} + \sum_{t=0}^{T} Z_{jt} F_t \qquad (5)$$

For present purposes we will drop the j's and assume that Z_t is a constant proportion L of APV_t, except that $Z_T = 0$. That is, it is assumed that the firm readjusts its debt level at the end of every period in terms of its value at that time, and that this level is maintained during the next period. Also we assume that MM are right, i.e., that $F_t = \dfrac{\tau r}{(1+r)^{t+1}}$. Thus

$$APV_o = A_o + \sum_{t=0}^{T-1} \frac{\tau r L (APV_t)}{(1+r)^{t+1}} \qquad (A.1)$$

Let $f = \dfrac{\tau r L}{1+r}$. Then

$$APV_{T-1} = A_{-1} + f(APV_{T-1})$$

$$= \frac{A_{-1}}{1-f} \qquad (A.2)$$

Having calculated APV_{T-1} we can determine APV_{T-2} from:

$$APV_{T-2} = A_{T-2} + f(APV_{T-2}) + \frac{f}{1+r}(APV_{T-1}) \qquad (A.3)$$

The general formula for any interim period $t - T-S$ is

$$APV_{T-S} = A_{T-S} + f(APV_{T-S}) + f \sum_{t=T-S+1}^{T-1} \frac{APV_t}{(1+r)^{t-T+S}} \qquad (A.4)$$

Of course Eq. (A.4) reduces to (A.1) when $S = T$.

This backwards-iteration procedure is tedious to work through manually, but it is not difficult to construct a computer program to do the calculations. Also, note that the calculations are done as a by-product of linear programming models of the sort mentioned in Section V above.

Proof that ρ^* is Independent of Project Profitability

Once APV_o is calculated for a project, then the true cost of capital ρ^* can be calculated via Eq. (8). But there is nothing evident in Eq. (8) that rules out the possibility of ρ^* being a function of the C_t's.

Nevertheless, it can be shown that ρ^* is independent of project profitability.

We can restate the cash flows in terms of a scale factor, and a pattern over time. That is, $C_t = \gamma_t C$ where $\gamma_1, \gamma_2, \ldots, \gamma_T$ are weights summing to 1. Also, let ρ_t^* be the true cost of capital for

the project at some intermediate point $1 \leq t \leq T - 1$.

I will now show that ρ^* (evaluated at $t = 0$) is independent of C.

We already know from Eq. (14) ρ_{T-1}^* is independent of C. Thus, consider ρ_{T-2}^* which is defined by

$$APV_{t-2} = C\left[\frac{\gamma_{T-1}}{1+\rho_{T-2}^*} + \frac{\gamma_T}{(1+\rho_{T-2}^*)^2}\right] \tag{A.5}$$

Substituting in Eq. (A.3),

$$C\left[\frac{\gamma_{T-1}}{1+\rho_{T-2}^*} + \frac{\gamma_T}{(1+\rho_{T-2}^*)^2}\right] = \frac{1}{1-f}\left[A_{T-2} + \frac{f}{1+r} \cdot \frac{\gamma_T C}{1+\rho_{T-1}^*}\right],$$

where

$$A_{T-2} = C\left[\frac{\gamma_{T-1}}{1+\rho_o} + \frac{\gamma_T}{(1+\rho_o)^2}\right].$$

Dividing through by C, we have an expression defining ρ_{T-2}^* in terms of ρ_{T-1}^* and the γ's, but not in terms of C.

Similarly, ρ_T^* can be defined in terms of ρ_{T-2}^*, ρ_{T-1}^* and the γ's. By working backwards we would eventually find that ρ^* evaluated at $t = 0$ is independent of C. It is also independent of C_o, the initial investment, since C_o is not discounted.

FOOTNOTES

1. Proposition I is stated in [17], p. 13. See [18] for MM's analysis of the impact of taxes.

2. There are cases in which market value is not an adequate proxy. Such cases may occur due to income taxes and transaction costs paid by investors, for example. The objective would have to be modified accordingly.

3. See [17], esp. p.   , Eq. (3); also [24], pp. 13-15.

4. See [15].

5. Note that this evaluates stock issues relative to the "base case" in which dividend policy is irrelevant. If dividend policy is irrelevant, given the firm's investment and debt decisions, then stock issues are also irrelevant to shareholders' wealth.

6. Some projects may be future investment opportunities anticipated for t=1, 2, ... Accepting such a project does not imply immediate investment, but simply that the project is included in the fimr's financial plan.

7. The limit may be imposed by capital markets or it may simply reflect management's judgment as to the best level of debt. For further discussion of the possible determinants of debt capacity, see Robichek and Myers [24], esp. pp. 13-22 and Baxter [1]. The literature on capital rationing is also relevant here. See, for example, Jaffee and Modigliani [9].

8. It can be argued that $\delta\psi/\delta D_t$ is negative, because dividends are
    taxed more heavily than capital gains. On the other hand, it is
    possible that some investors positively prefer dividends because
    of the convenience of having a regular, "automatic" cash income,
    or possibly for other reasons. For a summary, see Robichek and
    Myers [23a], ch. 4. Some authors believe that the various
    considerations cancel out, so that $\delta\psi/\delta D_t = 0$. See [16], pp.
    367-70, [2] and [6] for empirical evidence consistent with the
    irrelevance of dividend policy.

    In short, present evidence does not indicate that dividend policy
    is   all that important, apart from the "informational content"
    of dividends which is not germane here. Thus most of the analysis
    later in the paper assumes $\delta\psi/\delta D_t = 0$.

9. [16], p. 342. In MM's notation ρ^* is C(L) and ρ_{oj} is simply ρ_j.

10. Ibid., pp. 337, 340.

11. See Johnson [10], Ch. 11; Weston and Brigham [29], Ch. 11; Van
    Horne [28], Ch. 4.

12. Myers [21]. See also Mossin [19, 20], Hamada [8] and Levy and
    Sarnat [11]. Lintner [12] disagrees; he maintains that diversi-
    fication is an appropriate corporate objective.

13. Here is an example of complementary projects. Suppose that invest-
    ment options 1 and 2 are, respectively, a fleet of new trucks and
    a computer. If the trucks are purchased, then purchase of the com-
    puter will allow management to schedule usage of the trucks more
    efficiently. For this reason, the change in stock price if both
    projects 1 and 2 are accepted is greater than the sum of their
    present values separately considered. Note that this kind of inter-
    action has nothing to do with the risk characteristics of the vari-
    ous projects.

14. Eq. (11f) is not necesssary, since Eq. (11c) implies that project
    j will not change the risk characteristics of the firm's assets.

15. Alternatively, we could regard C as the expected value of the
    mean of the stream $\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_\omega$. See pp. 14-15 above.

16. Probably the most extensive and sophisticated test of the MM prop-
    ositions is MM's own study of the electric utility industry [16].
    This study supports their theory. There is controversy about MM's
    tests: see Robichek, McDonald and Higgins [23], Crockett and Friend
    [4],   Gordon [7], Brigham and Gordon [3] and Elton and Gruber [5].
    There clearly is room for a good deal more work, but despite the prob-
    lems, we can at least say that recent work is not inconsistent with
    the MM hypotheses. (See also Sarna and Rao [25] and Litzenberger
    and Rao [14].)

    The proposition of risk independence is even harder to test directly.
    There is circumstantial evidence indicating that diversification
    is not an appropriate goal for the firm -- for example, if investors
    were willing to pay for diversification would not closed-end mutual
    funds sell at a premium over asset value? And there is certainly
    no lack of diversification opportunities -- even the small investor
    can buy mutual funds.

    Tests of the "capital asset pricing model" of Sharpe [26], Lintner
    [13] and Mossin [19] may shed light on the risk-independence hypo-
    thesis. (The capital asset pricing model is sufficient but not
    necessary for risk-independence.) The empirical work to date indi-
    cates that the capital asset pricing model is probably an over-
    simplification, but it is too early to say for sure. Jensen [9a]
    reviews the theory and evidence.

17. MM, for example, were careful in [16] to note the limitations of
    their cost of capital analysis. One could also cite Tuttle and
    Litzenberger [27], Hamada [8] and many others.

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